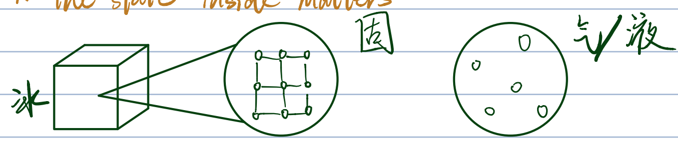
4. Thermal energy

4.1 Heat & temperature

4.1.1 Internal energy & temperature

I. Internal energy

1. the state inside matters



相同	分子种类一致 大量分子构成

不同	定点振动 分子间距小 有序排列	无规则运动 分子间距大 无序排列

2. internal energy & temperature

$EPE=\frac{1}{2}k\Delta x^2$; $KE=\frac{1}{2}mv^2$

internal E = EPE + KE

def. internal energy
the total energy (EPE+KE) stored in molecules of a body

	internal energy	temperature
联系	T↑. E↑	物体内部分子运动的体现
不同	微观	宏观

固: T↑ ↓ E↑ ↓ $\Sigma(\frac{1}{2}mv^2+\frac{1}{2}k\Delta x^2)$

气: T↑ ↓ E↑ ↓ $\frac{1}{2}mv^2$

ep. 温度↑但没有融化. 分子振动f↑

II. Units of T.

1. Celsius scale (°C) → 生活经验定义法

① 标准
ice melt → 0°C
water boils → 100°C

② 分度值

2. Kelvin scale / absolute temperature (K)

a. [illegible]标准
absolute 0 : 分子不振荡 (无法达到)

b. 分度值

temperature scale	Celsius (°C)	Kelvin (K)
water boiling point	100°C	373.15 K
ice freezing point	0°C	273.15 K
absolute 0	−273.15°C	0 K

T(K) = T(°C) + 273.15

4.1.2 Thermal energy & internal energy

I. Means of thermal energy producing (to increase internal energy)

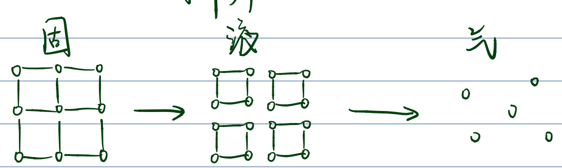
1.
- heating 热传递
  - conduction (接触)
  - radiation (辐射)
  - convection (流体对流)
  - 条件: $\Delta T \rightarrow \Delta E$
- work 做功
  - mechanics $\Delta E = F \cdot s$
  - electric $\Delta E = VI\Delta t$

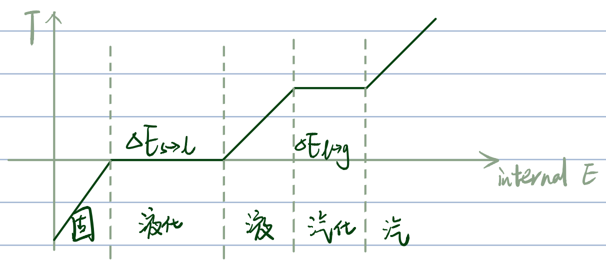
2. 辨析

	thermal	internal
联系	$E_{thermal}\uparrow$	$E_{internal}\uparrow$
不同	过程量	状态量

II. different states of material during internal energy↑ (absorbing heat)

液化 (组团解绑)





1. change between states (液化. 汽化. 升华)

①定性

T constant

thermal energy convert to potential energy of particle

gap of molecule↑

②定量

$\Delta E = mL$ (L = specific latent heat)

def. L: the energy required to change state of 1kg for an object in an constant temperature unit: $J\,kg^{-1}$

eg. $L_{水汽化} > L_{水液化}$

2. during a state (固. 液. 气)

①定性

T↑

all thermal energy convert to KE of particle
v↑ (gap 不变)

②定量

$\Delta E = P\Delta T = cm\Delta T$ (c = heat capacity)

δE 为 thermal energy

def. c: the energy required to increase 1kg body for 1K   unit: $J\,kg^{-1}\,K^{-1}$

c 由物质及物质的态决定

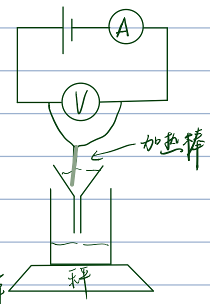
II. Experiments

1. measure the specific latent heat of fusion of ice

① $L = \frac{\Delta E}{m} = \frac{VI\Delta t}{m}$

② measurements: V. I. Δt. m

③ apparatus



④ plot a graph. 自变 - Δt. 因变 - m

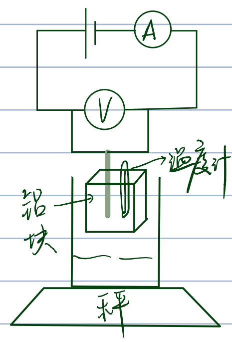
⑤ errors

漏斗与周围室温加快了冰的融化. $E_{实际} > E_{测}$ $L_{实际} > L_{测}$

同时读数　　录像/倒计时

碎冰掉落　　用带冰的滤纸

2. measurements the specific heat capacity of AL



① $C = \frac{\Delta E}{m\Delta T} = \frac{VI\Delta t}{m(T_f - T_i)}$

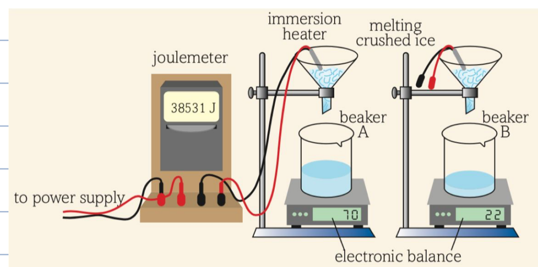
② measurement

③ apparatus

④ repeat reading T_{after} for different time

⑤ plot $\Delta T - VI\Delta t$

⑥ gradient = $\frac{1}{cm}$



⑦ errors

空气热传递效率较高　倒油 to improve the thermal contact

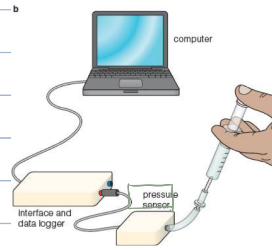
4.2 Gas laws & kinetic energy

4.2.1 Gas laws

n ← 单位体积内的分子数
V ← gap between molecules
T ← $\frac{1}{2}mv^2$
P ← F of collision of molecules ↑ ← { V↑ (体积), rate of collision ↑ ← v↑ (速度) }

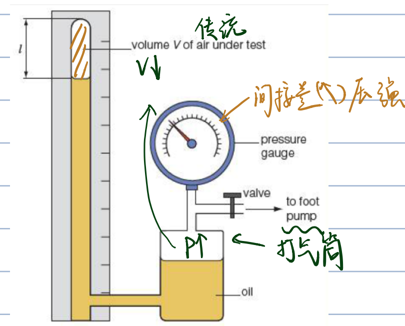
I. relation between T . p . V (Gas laws)

1. Boyle's — $p \propto \frac{1}{V}$ Constant T, M   控制变量



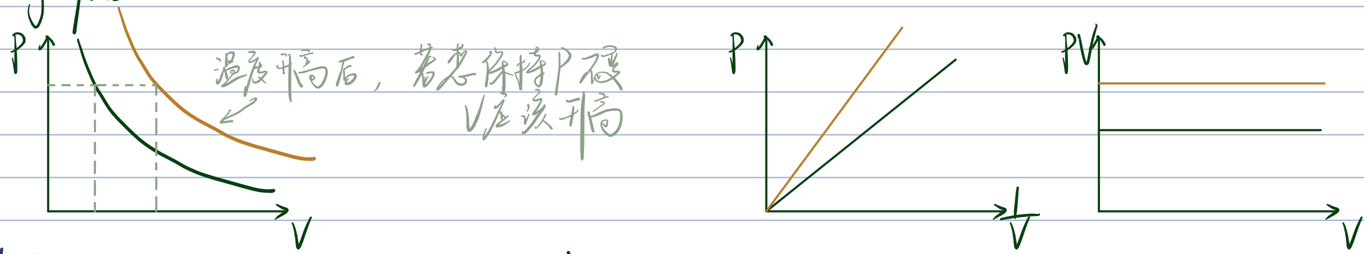
① 过程
constant syringe (针筒) with pressure sensor & data logger
change V slowly
record V in computer
repeat by changing V
plot a $p - \frac{1}{V}$ graph.
linear → $p \propto \frac{1}{V}$

$PV = C$



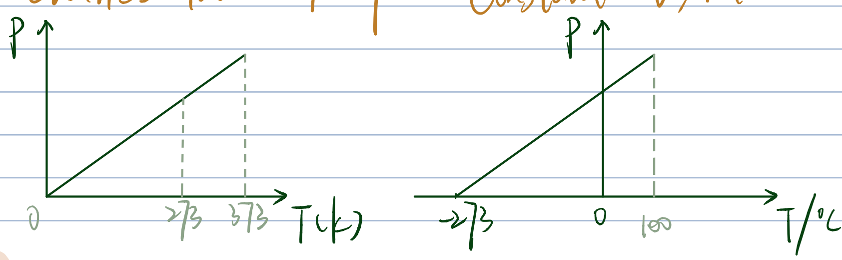
② 误差
缓慢推活塞 → keep T constant (做功会使 T↑)
针筒处有一段气体未被计算 → $V_{true} > V_{cal}$
large range (推不动 向外拉)

③ graphs



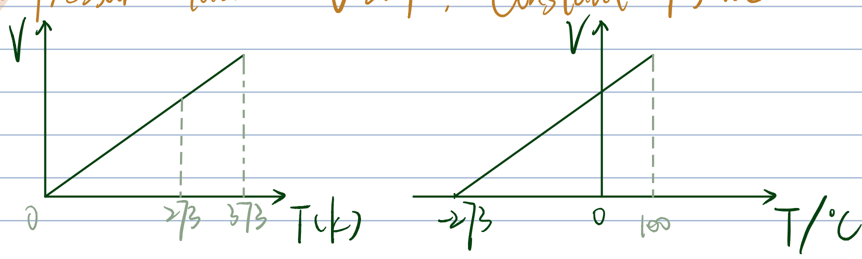
def. Boyle's law: for constant T, m, $P \propto \frac{1}{V}$

2. Charle's law — $P \propto T$ Constant V, m



$P \propto T$ $\frac{P}{T} = C$

3. Pressure law — $V \propto T$, Constant P, m



$T \propto V$

4. $PV \propto T$ when m is constant

$PV = kNT$ （条件：理想气体）

N: number of molecules

k: Bolzman constant $= 1.38 \times 10^{-23}\ m^2 kg s^{-2} K^{-1} = 1.38 \times 10^{-23}\ JK^{-1}$

R: universal molar gas constant

$Rn = kN$ $\quad R = 1.38 \times 10^{-23} \times 6.02 \times 10^{23} = 8.31\ JK^{-1}$

Ⅱ、理想气体状态方程 equation of state for ideal gas equation

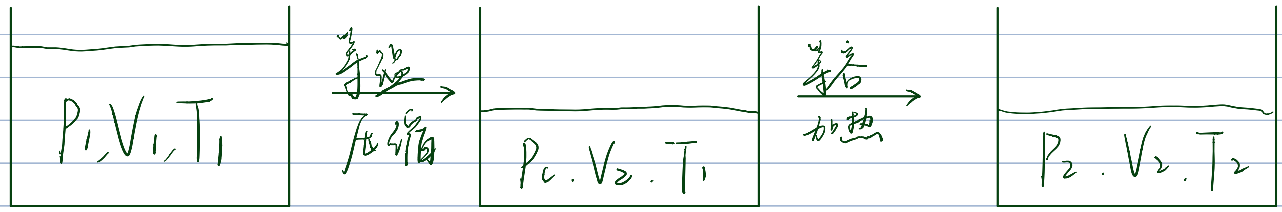
模拟理想气体
高温，低压

$PV = nRT = kNT$

ideal gas:
- 1. no size of molecules
- 2. no force between molecules except during collision
- 3. elastic collision of molecules
- 4. collision takes no time
- 5. large number of molecules
- 6. molecules are in continuous, rapid, random motion

→ 分子间作用力可忽略

推导



$P_1 V_1 = P_c V_2 \qquad \frac{P_c}{T_1} = \frac{P_2}{T_2}$

$$P_1 V_1 = \frac{P_2}{T_2} T_1 \cdot V_2$$

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

$$\frac{PV}{T} = nR = Nk$$

$$PV = nRT = NkT$$

$k = 1.38 \times 10^{-23}\ JK^{-1}$

$R = 8.31\ JK^{-1}$

N：分子数

n：(mol) 数

4.2.2 Kinetic theory – pressure of gas 分子动理论

microscopic property ← microscopic state / motion

- T ← kinetic energy of molecules (gas)
- V ← gap between molecules
- P ← collision ⊥ molecules

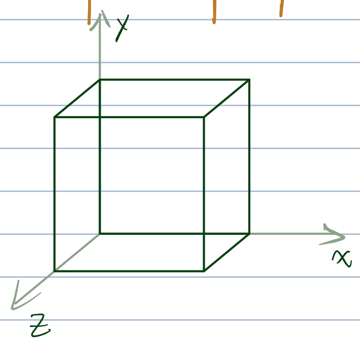
I. review of pressure

$$P = \frac{F}{A}, \quad P \perp A$$

1. 大小
   - ①固体 $p = \frac{F}{A}$
   - ②流体 $p = \rho g h$
2. 方向
3. 应用

II. Derivation of pressure

1. Equation for pressure based on kinetic theory



① Average force on the wall of molecule.

$$F_0 = \frac{\Delta p}{\Delta t} = \frac{2mv}{2l/v} = \frac{m v_x^2}{l}$$

$(\Sigma V_x = \Sigma V_y = \Sigma V_z)$

$$\overline{v^2} = \frac{\Sigma(V_x^2 + V_y^2 + V_z^2)}{3}$$

② The total force for all particles.

$$F = \frac{m}{l}(V_{1x}^2 + V_{2x}^2 + \cdots) = \frac{m}{l} \times \overline{v^2} \times N = \frac{mN\overline{v^2}}{3l}$$

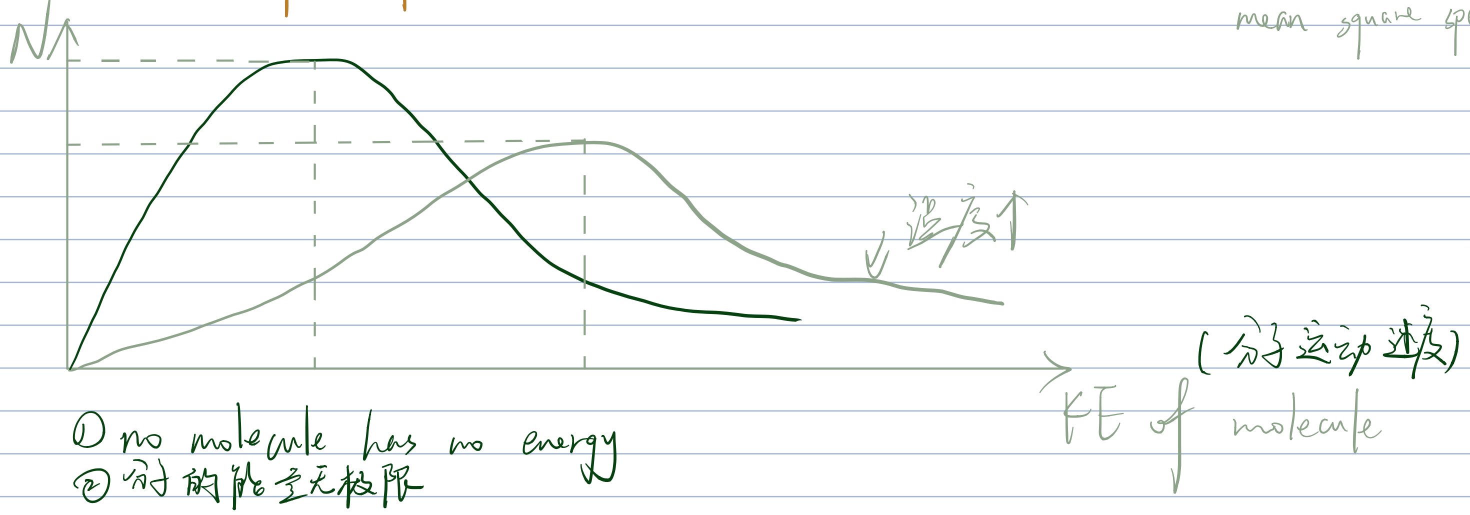
单位体积内的分子数

③ Pressure on this wall due to collision of molecules.

$$P = \frac{F}{A} = \frac{\frac{m}{l} \times N \times \frac{\overline{v^2}}{3}}{l^2} = \frac{Nm}{l^3} \times \frac{1}{3}\overline{v^2} = \frac{1}{3}\rho\overline{v^2}$$

mean square speed

2. Distribution of KE of molecules.



① no molecule has no energy
② 分子的能量无极限

3. Relationship between KE & T.

$$PV = NkT$$
$$\frac{1}{3}\rho\overline{v^2}V = NkT$$
$$\frac{1}{3}M\overline{v^2} = NkT$$
$$\frac{1}{3} \times \frac{M}{N} \times \overline{v^2} = kT$$
$$\frac{1}{3}m\overline{v^2} = kT$$

$$KE = \frac{1}{2}m\overline{v^2} = \frac{3}{2}kT$$

$$\overline{KE} \propto T$$

gap → V ↘ P
$\overline{KE}$ → T ↗ P

温度是微观物质分子的平均动能，反映分子运动的剧烈程度

4. Express pressure law by kinetic theory of gas

① constant T, $V \propto \frac{1}{P}$ $V\uparrow \rightarrow P\downarrow \rightarrow \frac{\Delta n_{collision}}{\Delta t}\downarrow \rightarrow$ Pressure $\downarrow$

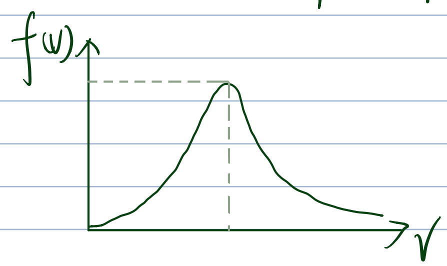
② constant V, $P \propto T$ $T\uparrow \rightarrow KE\uparrow \rightarrow \bar{v}\uparrow \rightarrow \Delta p\uparrow \rightarrow$ Pressure $\uparrow$
(动量)

③ constant P, $T \propto V$ $T\uparrow \rightarrow KE\uparrow \rightarrow \bar{v}\uparrow \rightarrow \Delta p\uparrow \rightarrow$ Pressure $\uparrow \rightarrow$ ΣP constant
$V\uparrow \leftarrow P\downarrow \leftarrow \frac{\Delta n_{collision}}{\Delta t}\downarrow \leftarrow$ Pressure $\downarrow \leftarrow$

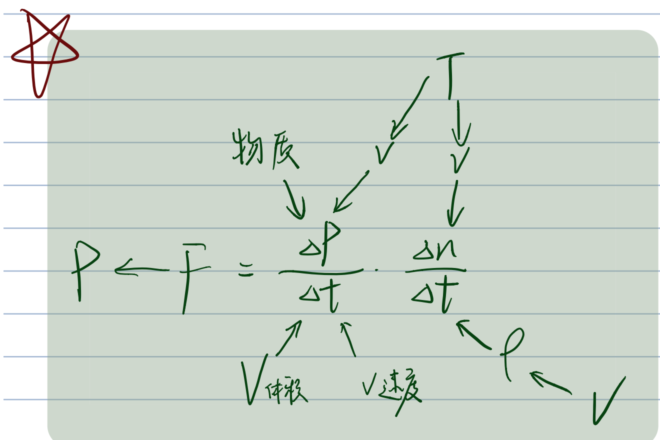
$\overline{v^2}$ mean square speed

$\sqrt{\overline{v^2}}$ root mean square speed (r.m.s) $= \sqrt{\frac{3kT}{m}} = 1.73\sqrt{\frac{kT}{m}}$

(分子间作用力)
$\vec{F} \rightarrow E_p$



↑
拥有most probable speed



4.1

④ temperature (外在体现) ∝ $\overline{KE}_{molecules}$

↑ 仅相同状态下为线性关系

③ internal energy (分子的内在属性) = Σ(KE + PE)

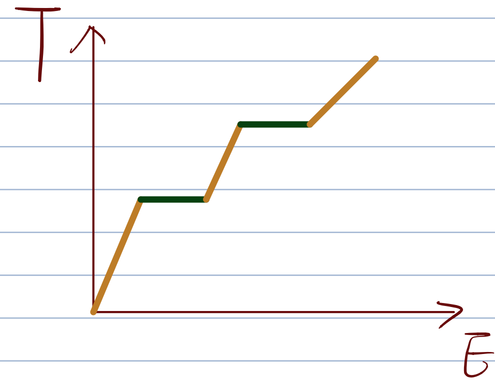
- KE ↑ T; KE ↑ v; $\Delta E = mc\Delta T$
- PE ↑ V (体积); PE ↑ 分子 gap; $\Delta E = mL$

↓ thermal energy 是内能

② thermal energy (W)

↓

① heating { conduction, radiation }
   work { mechanical, electric }



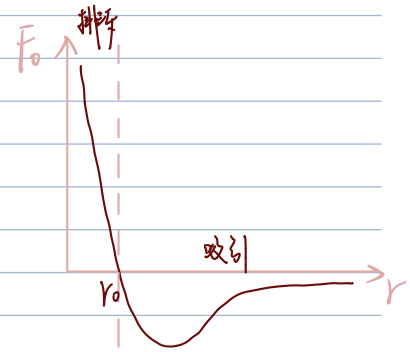
微观上分子状态决定宏观量 eg. 温度. 体积

T → $\overline{KE}$ molecule ↘ Internal E↑

V↑ (体积) → gap between molecule↑ → PE molecule↑ ↗

Macro property is based on micro state molecules

1. for all states:



- $\overline{KE}$ of molecules $\rightarrow$ T: $\frac{3}{2}kT = \frac{1}{2}m\langle c^2\rangle$
- T — thermal energy: $\Delta E = cm\Delta T$
- thermal energy $\rightarrow$ internal energy: $\Delta E = E_2 - E_1$
  - heating { conduction, radiation }
  - work { mechanical, electric }
- internal energy $\rightarrow$ $\overline{KE}$ of molecules: during states
- internal energy $\rightarrow$ $\overline{PE}$ of molecules: change states
- $\overline{PE}$ of molecules $\rightarrow$ V
- thermal energy — V: $\Delta E = mL$

2. Only for gas

micro-experiment: gas laws for fixed mass ideal gas,
- Boyle's law. constant T. $P \propto \frac{1}{V}$
- Pressure law constant V $P \propto T$
- Charle's law constant P $T \propto V$

ideal gas:
① no size
② no force
③ elastic collision
④ no time
⑤ large N
⑥ random motion

ideal gas states: $PV = NkT$, $PV = nRT$

micro-theory: kinetic theory of ideal gas $\longrightarrow$ $P = \frac{1}{3}\rho\langle c^2\rangle$ $\longrightarrow$ $\frac{3}{2}kT = \frac{1}{2}m\langle c^2\rangle$

5. Nuclear reaction

质子+中子 → $^{A}_{Z}X$ (Z ↓ proton)

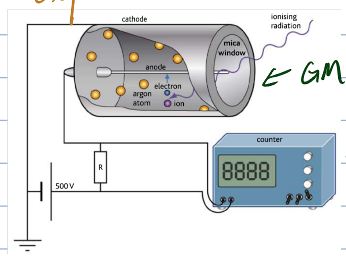
5.1 Nuclear decay 衰变

5.1.1 Nuclear radiation

- 电离辐射 ionizing radiation ep. X-ray, γ-ray, β, α
- 非电离辐射 nonionizing radiation ep. micro~, UV, radio~, IR

I. Activity of radiation

1. Experiment



← GM tube

← counter

↓

仅显示通过的粒子数

count rate $= \frac{\Delta N}{\Delta t}$ 越高辐射越强 reflect light be photoed

G-M tube

① high energy particle will ionize argon atom in tube

② E ↓ e accelerate to anode & ionized to cathod

测电离次数 = 反应辐射的量 ≠ 辐射粒子的量

2. Activity rate of ionization

unit: count per second (Bq)

☆(题之不定时) 应测两次, 存在 background radiation

$$1u = 1.66 \times 10^{-27} kg = 931.5 MeV/c^2$$

II. Background radiation

1. dose
2. sources
3. how to improve accuracy
4. significance
   deform DNA { 好处: important in evolution 进化; 坏处: cause cancer }

III. How nuclear radiation produced?

(What kind of nuclei will decay?)

1. α-decay (带电量最大)

$$^{241}_{95}Am \longrightarrow ^{237}_{93}Np + ^{4}_{2}He$$

2. β-decay

$$^{90}_{38}Sr \longrightarrow ^{90}_{39}Y + ^{0}_{-1}e$$

根:$n \rightarrow p + e$

3. γ-decay (excited) 伴随α与β出现. 不带电

$$^{60}_{28}Ni\text{-}m \longrightarrow ^{60}_{28}Ni + ^{0}_{0}\gamma$$

IV. How to quantitatively describe the strength of radiation?

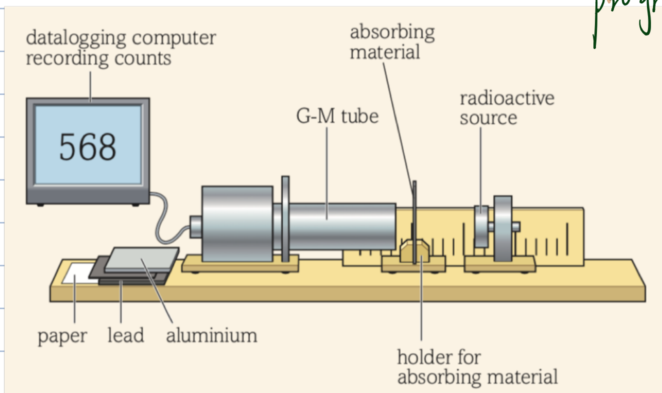
1. two quantities
   ① ionizing ability
   ② penetration (range of radiation)

ionizing ability 越强, penetration 距离越短

2. How to investigate penetration by G-M tube

① for different medium

progressively increase in density. air, paper, Al, lead



medium	α	β	γ
air	✓	✓	✓
✓ paper	✗	✓	✓
✓ 5mm Al	✗	✗	✓
10 cm lead	✗	✗	✗
穿透性	弱	→	强
电离性	强	→	弱

γ → 治疗癌症

* explain the penetration of α, β, γ based on their nature property

Q ↑ → $\frac{N_{ionization}}{\Delta t}$ ↑ → $\frac{\Delta KE}{\Delta t}$ ↑ → distance when all energy is loss ↓

② investigate the absorption of γ by lead

experiment: penetration of γ in lead

1. measurement { thickness of lead pieces; count rate

Property	α-particle	β-particle	γ-radiation
mass	4 u (6.64 × 10^{-27} kg)	about $\frac{1}{2000}$ u (9.11 × 10^{-31} kg)	0
charge	+2e	−e	0
speed	up to about $\frac{1}{20}c$	up to about 0.99c	c
typical energy	0.6 to 1.3 pJ (4 to 8 MeV)	0 up to 2.0 pJ (0 to 12 MeV)	0.01 to 1.0 pJ (0.06 to 6 MeV)
relative ionising power	10 000	100	1
penetration	few cm of air	few mm of aluminium	few cm of lead 或几米的 concrete wall
deflection in electric and magnetic fields	small deflection	large deflection	no deflection
nature	helium nucleus	electron	high-frequency electromagnetic radiation

2. apparatus {
- vernier caliper
- G-M tube, counter, stopwatch
- lead paper with different thickness
- γ source (用α.β纸过滤α)

3. process
① measure background count rate. 制做γ source
② 先测一组无lead, count rate
③ 改变lead厚度. record 相同 time period 的 count rate
④ correct count rate → 测量差 - background
⑤ 画图 correct count rate → 原 - thickness

$y = \ln\frac{A}{A_0}$ $x = t$

V. application: the hazard levels to human body for each type

	type of radiation	Inside body	Outside body
^4_2He 原子核	α	danger – ionising 强	no danger – penetration 弱
电子	β	danger – ~ moderate	danger – ~ moderate
能量	γ	danger if long period	danger if long period

5.1.2 Radioactive decay

I. Nature of radioactive decay

1. Spontaneous ← 不能改变

not be influenced by other factors

2. Random ← 不能预测

不确定：{ n个核：which nuclei will decay in next second
1个核 when will a particular nucleus decay

确定：there is a certain probability that a nucleus will decay for each second

↑ decay constant (λ)

一个原子核一秒内decay的概率 unit: s^{-1}

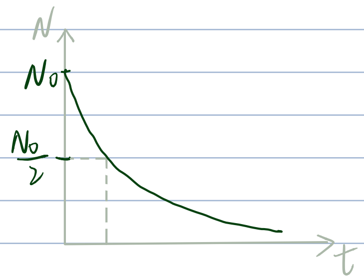
II. Mathematical treatment

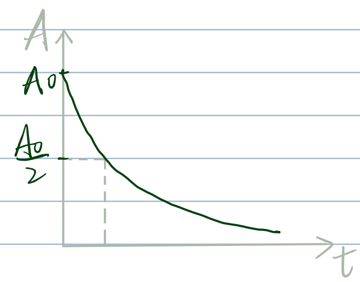
1. $A = N\lambda$ $A = -\frac{dN}{dt}$

↓

$N = N_0 e^{-\lambda t}$

$A = A_0 e^{-\lambda t}$





2. 半衰期 ($t_{\frac{1}{2}}$)

$\frac{N_0}{2} = N_0 e^{-\lambda t}$

$\frac{1}{2} = e^{-\lambda t}$

$-\lambda t = \ln\frac{1}{2}$

$t_{\frac{1}{2}} = \frac{\ln\frac{1}{2}}{-\lambda} = \frac{\ln 2}{\lambda}$

$t_{\frac{1}{2}} = \frac{\ln 2}{\lambda}$

$t = n \times t_{\frac{1}{2}}$

↑ 从始到末有几个半衰期

$t = \log_{\frac{1}{2}}\left(\frac{N}{N_0}\right) t_{\frac{1}{2}}$

λ: decay probability for 1 nucleus in 1s

A: $\frac{\text{number of decay}}{\Delta t} = \frac{\Delta N \text{ of radioactive nuclei}}{\Delta t}$

$A = N\lambda$ { $N = N_0 e^{-\lambda t}$, $A = A_0 e^{-\lambda t}$ } unit A: Bq

$t_{\frac{1}{2}}$: time when N of radioactive nuclei decrease to its half

III. 测半衰期实验

Determine nature constant of radioactive nuclei by experiment

问题：如何用实验的方法测量放射性物质的半衰期以及 decay constant

① measurement { A_b: background radiation
t
A

② apparatus { stop watch, G-M tube . counter

③ process of data & get results

④ plot a $\ln A - t$ graph. gradient $= -\lambda$

$A = A_0 e^{-\lambda t}$ $\ln A = -\lambda t + \ln A_0$

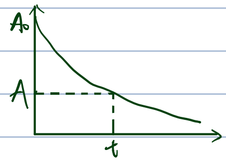
IV. Application of radioactive decay

1. radioactive decay

① C 14 dating in archaeology

$A = A_0 e^{-\lambda t}$

$\frac{A}{A_0} = \left(\frac{1}{2}\right)^{\frac{t}{t_{1/2}}} \rightarrow \begin{Bmatrix} A \\ A_0 \end{Bmatrix} \rightarrow t$



a. $\frac{C_{14}}{C_{12}}$ is constant in atmosphere

(C_{14} live plants = atmosphere: constant ; die → decay)

b. $t_{\frac{1}{2}} = 5730$ years. (不能测年代过久或年代过近的)

2. P-40 dating in geology

① $t_{\frac{1}{2}} = 1.3 \times 10^9$ years (可测地球年龄)

① diagnosic imaging

② therapy

a. radiation ionizing ability

γ is danger only for long time explosion

b. $t_{\frac{1}{2}}$ { 长, 短

Isotope	Emission	Energy	Half life	Use
Technetium-99m (^{99m}Tc)	γ	140 keV	6.0 h	*Diagnosis:* Localisation of tumours; Monitoring blood flow in heart and lungs; Kidney investigations
Iodine-123 (^{123}I)	γ	160 keV	13 h	*Diagnosis:* Localisation of tumours; Assessing thyroid function
Iodine-131 (^{131}I)	β^-, γ	360 keV	8 days	*Therapy:* Thyroid function and tumours

(β^- →) 杀伤癌细胞

5.2 Nuclear reaction

5.2.1 Nuclear binding energy

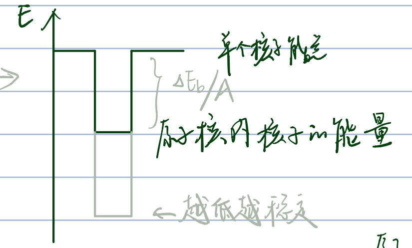
I. Mass deficit

$\Delta m = x m_p + y m_n - m_A$ $\Delta E = \Delta m c^2$

def. mass deficit: the mass loss when nucleons combine to be a nucleus.

II. Nuclear binding energy (主语：原子核)

mass deficit convert to some form of energy



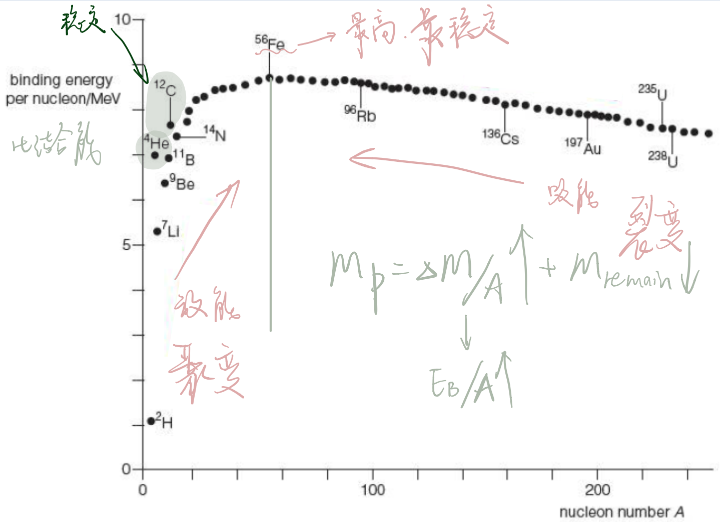
Ex. C_{12} in E_B in eV

$\Delta E = \Delta m c^2 = 0.0989 \times 1.66 \times 10^{-27} \times (3\times10^8)^2 = 1.48\times10^{-11} J = 9.25\times10^7 eV$

def. binding energy: ① the energy released when nucleons (核子) combined together to be a nucleus (原子核).
② energy needed to seperate nucleus as individual nucleons.

III. Binding energy per nucleons 核子释放的能量

$= \frac{E_B}{A}$ ← number of nucleons



质量增加 → 吸热
质量减少 → 放热

p.n → nucleus

反应向更稳定的进行

$M_p = \frac{\Delta M^{(M_B)}}{A} + M_{reaction}$

5.2.2 Nuclear fusion and fission

I. Nuclear fusion 核聚变

1. def. 2 light nuclei join to make a single nucleus　　质量丢失，释放能量
   The average binding energy per nucleon increases, release energy and loss mass.

2. 典型反应: $4\,^1_1H \longrightarrow {}^4_2He + 2\,^0_1X + 2\gamma$
   配平　　*不存在 ^0_1X

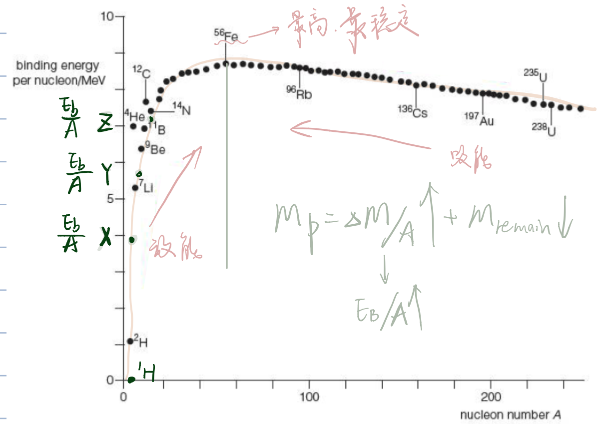
3. 反应过程: ① $2\,^1_1H \longrightarrow {}^2_1H + {}^0_1e + \gamma$
   ② $^2_1H + {}^1_1H \longrightarrow {}^3_2He + \gamma$
   ③ $2\,^3_2He \longrightarrow {}^4_2He + 2\,^1_1H + \gamma$

4. 发生条件　　核子间存在静电力，需克服核与核的静电力，若距离 $< 10^{-15}$，才达到强合力作用范围
   ① 微观: $d < 10^{-15}\,m$
   ↑ KE　overcome electrostatic repulsive force ↗

   ② 宏观
   - *高温: KE核↑ → 克服静电力　$10^7\,K$
   - 高压: 单位时间内碰撞次数↑ ($\frac{\Delta n}{\Delta t}$)　$10^{16}\,Pa$

   ? 为何在 Sun 中可操作，但在地球上难
   Sun ① $T\uparrow \rightarrow \frac{1}{2}m\langle v^2\rangle \rightarrow$ overcome repulsive force → d足够小
   ② $P\uparrow \rightarrow \frac{\Delta n}{\Delta t}\uparrow$ 以致碰撞次数↑
   ③ high gravitational force
   Earth ① container { melt / fusion cease
   ② 安强磁场 → 但 hard to reach 高电流

5. 应用
   ① stellar fusion
   long time working gravitational forces
   hot, dense material is held together by gravitational force
   ② Fusion on Earth
   磁场，垂直作用



6. 计算核反应放能
   $^{a+b}_{a}X + Y \longrightarrow Z$
   mass loss $\Delta m = m_X + m_Y - m_Z$　　mass deficit = mass loss (p+n → nucleus)
   　　$\times c^2\downarrow$　　$\times c^2\downarrow$
   mass deficit $m_{X\,deficit} = am_p + bm_n - m_X$　　mass energy = released energy (p+n → nucleus)

   *① mass loss $\times c^2 \longrightarrow$ energy released
   ② 如图 $X\left[\left(\frac{E}{A}\right)_Z - \left(\frac{E}{A}\right)_X\right] + Y\left[\left(\frac{E}{A}\right)_Z - \left(\frac{E}{A}\right)_Y\right]$

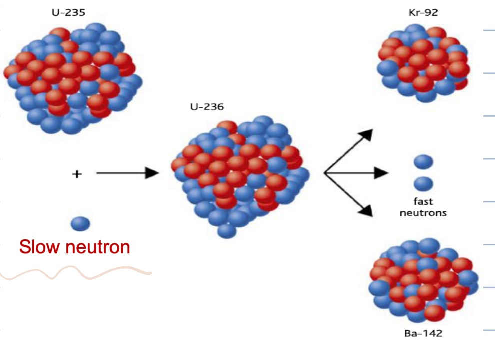
Ⅱ. Nuclear fission 核裂变

1. def.

A heavy nucleus splitting to form 2 lighter nuclei.
The average binding energy per nucleon increases. → release energy → mass loss

2. 典型反应

① 反应结果 A↑, n/p比 → 放出n



② 反应条件
加低速中子

slow 链式反应 fast

③ 反应过程

$$^{235}_{92}U + ^{1}_{0}n \rightarrow ^{92}_{36}Kr + ^{142}_{56}Ba + 2^{1}_{0}n + energy$$

$$^{235}_{92}U + ^{1}_{0}n \rightarrow ^{92}_{36}Kr + ^{141}_{56}Ba + 3^{1}_{0}n + energy$$

3. 应用（比聚变更广，放能更大）

① 为什么广泛

条件 { fusion：极高温 高压
　　 { fission：slow neutron

② 反应特点 1 slow n vs. 2-3 fast n

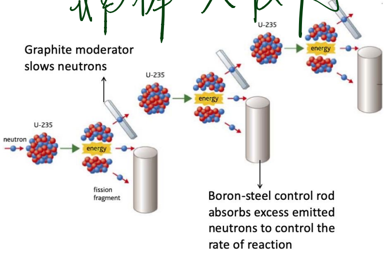
反应产物可作为下一个反应的条件

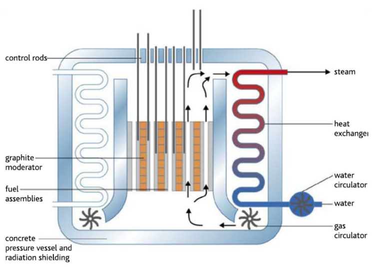
- chain reaction 可行性
- chain reaction 前提：给产物中子减速
- 反应速度会指数性增加

③ 启发应用

- 减速 → 产物中子
  moderator → slow down the fast neutron
- 减速 → 反应速度
  控制反应条件中子的数量

镉棒 吸收中子





radiation ⟵ Decay

background radiation → α . β . γ nature → ionizing ability ↔ penetration/range

Q. KE → $\left(\frac{\Delta n}{\Delta t}\right)$ → $\frac{\Delta KE}{\Delta t}$ → $S_{KE} \to 0$

nature { spontaneous, random

→ λ: decay probability, 1 nucleus/1s.

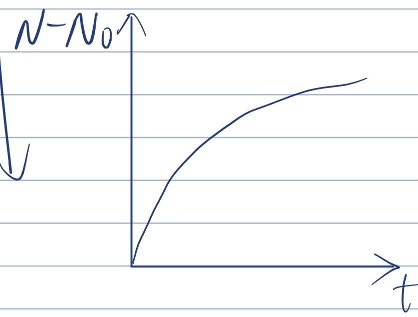
→ A: N of decay/1s

$$-\frac{dN}{dt} = A = N\lambda$$

$$N = N_0 e^{-\lambda t} \longrightarrow \frac{N}{N_0} = \left(\frac{1}{2}\right)^{\frac{t}{t_{1/2}}}$$

$$A = A_0 e^{-\lambda t} \longrightarrow \frac{A}{A_0} = \left(\frac{1}{2}\right)^{\frac{t}{t_{1/2}}}$$

$t_{\frac{1}{2}}$: time/N/A decrease to its half → $t_{\frac{1}{2}} = \frac{\ln 2}{\lambda}$



experiment: penetration of γ in lead

1. measurement { thickness of lead pieces, count rate

2. apparatus { vernier caliper, G-M tube · counter, stopwatch, lead paper with different thickness, γ source

3. process

★核反应遵循电荷数&质量数守衡，非质量守衡

	fusion 聚	fission 裂	哪个好
条件	极高温高压	slow neutron	fission
放能	6MeV/nucleon	0.7MeV/nucleon	fusion
原物	unlimit supply	有限 (U is limited)	fusion
生成物	^4_2He 无害	radioactive waste	fusion

{ MeV 能量单位, MeV/c^2 质量单位

Nucleus persue more stable by reaction

结构不稳 → radioactive decay

p/n 占比 (n+p越多 n占比越大越不稳定)

↓

nuclear radiation

Q.m.KE = α.β.γ nature

count rate $(\frac{\Delta n}{\Delta t})$ = ionising ability

d = penetration / range

不确定 { spontaneous: can't be influenced
random: can't tell ...

确定 { λ: decay probability /nucleon/s
A: decay number/s

$$-\frac{dN}{dt} = A = N\lambda$$

$$N = N_0 e^{-\lambda t} \longrightarrow \frac{N}{N_0} = \left(\frac{1}{2}\right)^{\frac{t}{t_{1/2}}}$$

$$A = A_0 e^{-\lambda t} \longrightarrow \frac{A}{A_0} = \left(\frac{1}{2}\right)^{\frac{t}{t_{1/2}}}$$

$t_{1/2}$: time for N/A decrease to its half

能量不稳 → mass deficit = mass loss (p.n → nucleus)

↓ $\times c^2$ ↓ $\times c^2$

binding energy = released energy (p.n → nucleus)

−E of proton

−E of nucleon in x

−E of nucleon in y

① y: B/A ↑ → energy level ↓ → 稳定

② x → y: B/A ↑ → release E → loss m

聚变 & 裂变

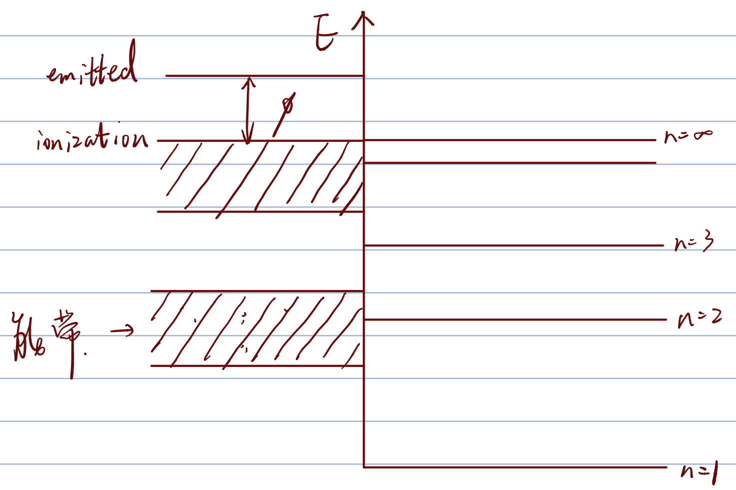
同 放能 异

two light → one   one heavy split two

条件 { 微: KE↑ $\frac{\Delta n}{\Delta t}$↑
宏: T↑ P↑

条件 { slow n
fast n
chain reaction

固体 气体



微观粒子：能量不连续，且越低越稳定

6. Oscillation

6.1. Simple harmonic motion

6.1.1 SHM kinetic model

I. def.

位移与t关系为正弦函数

Restoring force (合力)

① 回复力大小与位移成正比

大小: Resultant force act on the oscillation body which proportional to the displacement of the body. (当ΣF=0时)

② 与位移方向相反

方向: the resultant force always towards equilibrium position

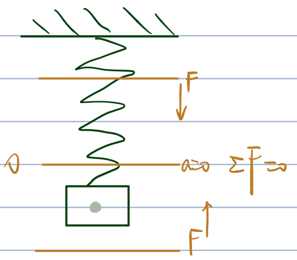
$\Sigma F = -kx$ ← 相对平衡位置的位移

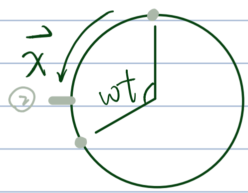
$a = -\frac{kx}{m} = \omega^2 A$

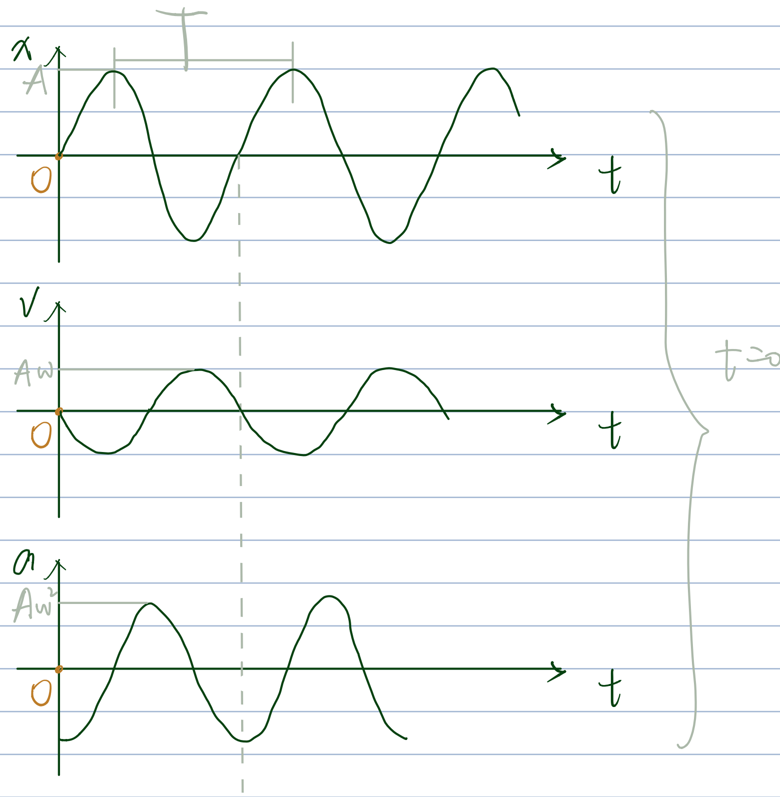
restoring force (区别弹力与回复力)

force constant $\begin{cases} k\uparrow, x\downarrow \\ k\uparrow, \Sigma F\uparrow \end{cases}$

II. Motion







$\Sigma F = -k\Delta x$

$\Delta x = A\cos\omega t$ ⟶ constant T

位置 $x = A\sin(\omega t + \varphi)$ (振幅 A, 角速度 ω, 初相位 φ)

速度 $v = \Delta x(t)' = -A\omega\sin(\omega t + \varphi)$

加速度 $a = \Delta x(t)'' = -A\omega^2\cos(\omega t + \varphi)$

$\vec{a} = -\omega^2\vec{x}$

$\Sigma\vec{F} = -k\vec{x}$ ⟶ $\omega^2 = \frac{k}{m}$ ⟶ $\omega = \sqrt{\frac{k}{m}}$

$\vec{a} = -\frac{k}{m}\vec{x}$

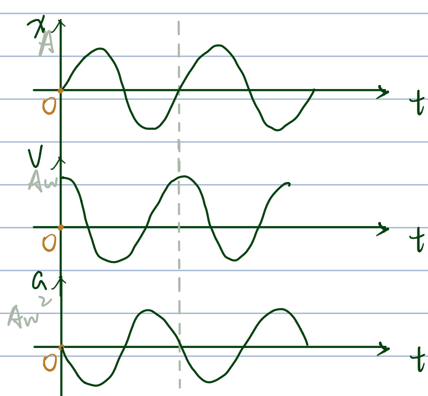
$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$

t=0, x=0, 向上:

$x = A\sin(\omega t)$

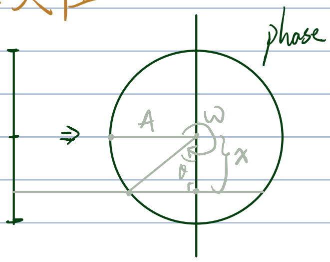
$v = -A\omega\cos(\omega t)$

$a = -A\omega^2\sin(\omega t)$

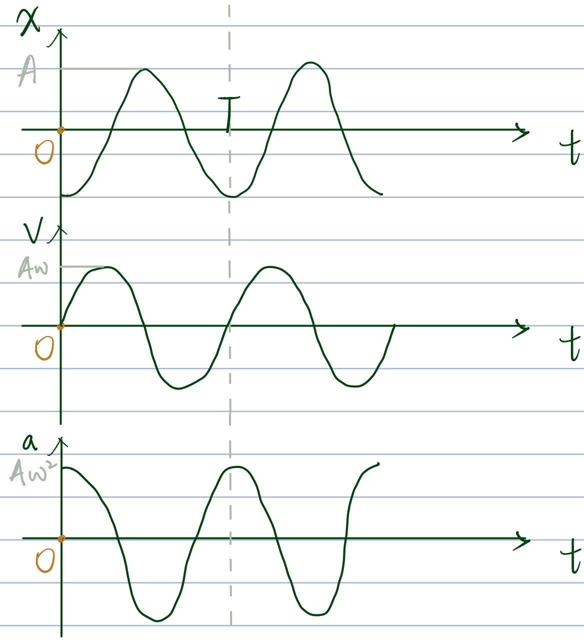


Ⅲ. 建模

1. 定性



$\theta = \omega t$



2. 定量

for $t=0$ $\quad x=-A$

$x = -A\cos\theta = -A\cos(\omega t)$

$v(t) = x' = A\omega\sin(\omega t)$

$a(t) = x'' = A\omega^2\cos(\omega t)$

① $x_{max} = A$

$V_{max} = A\omega$ $\quad \to \sum F_{max}$ 为最高与最低点 $= kA$ $\xrightarrow{\sum F \propto x}$ x_{max}

$a_{max} = A\omega^2$

$x = A \to \sum F = kA \to a = \frac{k}{m}A = \omega^2 A$

② $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$ nature property

思考：在太空中如何称重 

③ 改变起点，什么变，什么不变？

变：$x(t)$，$v(t)$，$a(t)$，φ

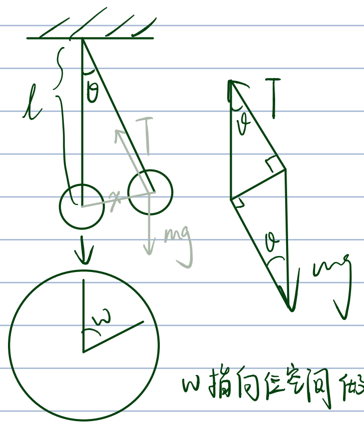
*不变：$\omega = \sqrt{\frac{k}{m}}$，$T = 2\pi\sqrt{\frac{m}{k}}$，$x_{max}$，$V_{max}$，$a_{max}$

6.1.2. Pendulum → SHM

I. 验证是否为 SHM

$\Sigma F \propto x$

假设 { string：light，无形变；小球：massive，small }



$\sin\frac{\theta}{2} = \frac{x}{2} \times \frac{1}{l} = \frac{x}{2l}$

$x = 2l\sin\frac{\theta}{2}$

→ $\Sigma F = mg\sin\theta$

$x = 2l\sin\frac{\theta}{2}$

$\theta \approx 0$ ↓ $\theta = \sin\theta = \tan\theta$

$\Sigma F = mg\theta$

$x = l\theta$

↓

$\Sigma F = -\frac{mg}{l}x \quad (\theta \sim 0)$

↓

小角度单摆，为 SHM　　$k = \frac{mg}{l}$

ω 指向任意时间做匀速圆周运动 in ω

$\omega = \frac{2\pi}{T}$

So pendulum is SHM,
for small angle ($< 10^\circ$)

测 v_{max} & a_{max}：　$\omega = \sqrt{\frac{g}{l}}$

$v_{max} = A\omega \quad a_{max} = A\omega^2$ / 能量守恒

II. Period of pendulum.

$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{\frac{mg}{l}}} = 2\pi\sqrt{\frac{l}{g}}$

⊕ 单摆周期与 m、振幅无关

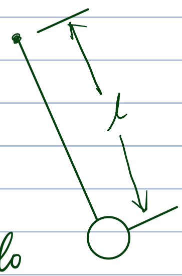
III. 实验：用单摆测 g

1. process

① measurements — apparatus

l = length of string $l + \frac{\text{diameter}}{2}$

T — stopwatch



② repeat measure (l_0 & T) for different l_0

③ process the data

$x \to l = l_0 + \frac{d}{2}$　　$y \to T^2$

④ plot y–x graph → best fit line, determine gradient

⑤ $g = \frac{4\pi^2}{\text{gradient}}$

2. How to improve accuracy

① small angle ($\theta < 10^\circ$) → constant T

② repeat measure d at different orientations and take average → uniform → random error↓

③ long string → T↑ → %U↓

④ { string: m↓; ball: m↑ 小 }

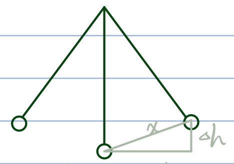
⑤ fiducial point at equilibrium position

⑥ $NT = t$　　$\%U = \frac{U}{\text{value}}$

6.1.3 SHM energy

I. Energy transfering

1. 定性



$\Delta h = x\sin\alpha = x\sin\frac{\theta}{2}$

$\Delta h = \frac{x\theta}{2} = \frac{x^2}{2l}$

$\theta = \frac{x}{l}$

v	0	$\pm max$	0
$\frac{1}{2}mv^2$	0	max	0
x	$-max$	0	$+max$
mgh	max	0	max

2. 定量 — general equation

$k = \omega^2 m$ (Nm^{-1})

$PE = \frac{1}{2}kx^2$ $\quad PE = mg\Delta h = \frac{mg}{2l}x^2 = \frac{1}{2}kx^2$

$KE = \frac{1}{2}mv^2 = \frac{1}{2}m(A\omega\sin(\omega t))^2 = \frac{1}{2}mA^2\omega^2\sin^2(\omega t)$

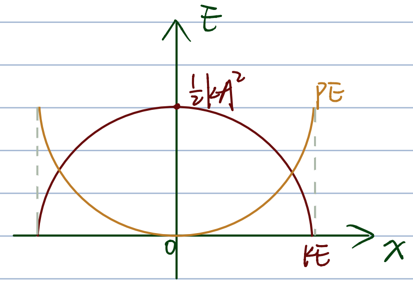
$= \frac{1}{2}mA^2\omega^2(1-\cos^2\omega t)$

$= \frac{1}{2}mA^2\omega^2 - \frac{1}{2}m\omega^2x^2$ $\quad \omega = \frac{k}{m}$

$= \frac{1}{2}kA^2 - \frac{1}{2}kx^2$

$KE = \frac{1}{2}kA^2 - \frac{1}{2}kx^2$

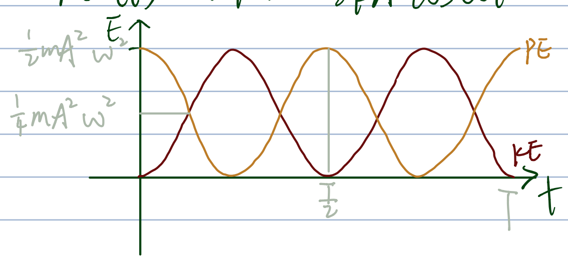
$E_{total} = \frac{1}{2}kA^2$ → constant (同时也为 KE_{max} 与 PE_{max})



II. Energy change with time → E(t)

$KE(t) = \frac{1}{2}mA^2\omega^2\sin^2\omega t = \frac{1}{4}mA^2\omega^2(1-\cos 2\omega t)$

$PE(t) = \frac{1}{2}kx^2 = \frac{1}{2}kA^2\cos^2\omega t$



$E_{total} = \frac{1}{2}kA^2$

6.2 Oscillations in real life

6.2.1 Damped and forced oscillation

SHM model.(理想模型) $W_{F_{ex}}=0$ E is constant. $f_0 \propto \sqrt{\frac{k}{m}}$ — free oscillation

外力作功

I. Damped oscillation

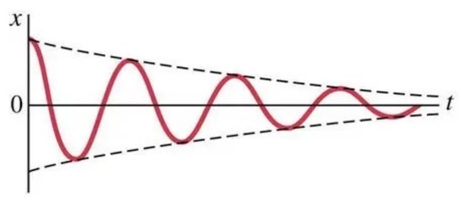
1. 实例 bungee 蹦极

问1. 特点

① x轴: constant T

② y轴: A↓ exponentially

Damped harmonic motion is harmonic motion with a frictional or drag force. If the damping is small, we can treat it as an "envelope" that modifies the undamped oscillation.



问2. 为什么A↓?

$f \xrightarrow{-W} E\downarrow \xrightarrow{E=\frac{1}{2}kA^2} A\downarrow$

问3. T由什么决定. T constant 合理吗?

$T=\frac{2\pi}{\omega}=2\pi\sqrt{\frac{m}{k}}$ mass of oscillation, force constant

$F=-kx$

2. 定义和特点

① def. opposite force on body, eg. f

↓ −W

E dissipated into thermal

↓

A decrease with time

} damped

② 特点 constant $f=\frac{1}{2\pi}\sqrt{\frac{k}{m}}$, determined by force constant of system and mass of osscication

$k=\frac{mg}{l}$

$\Sigma F=-\frac{mg}{l}\vec{x}$

we call it natural frequency

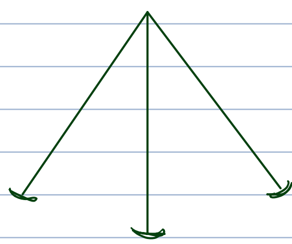
II. Forced oscillation

1. 实例: 秋千

问1. 如果没人推, 他会做什么运动?

考虑阻力: damped oscillation −f

理想阻力: free oscillation



问2. 如果有人给他推力. 秋千可能怎么运动

A↑/A↓

问3. 会推的小伙伴是怎么推的? 恒力

周期力. $T_{力}=T_{motion}$

2. 概念

① Forced oscillation & driving frequency

forced oscillation: system oscillates under the influence of an external repeated force

driving frequency: the frequency of the external force

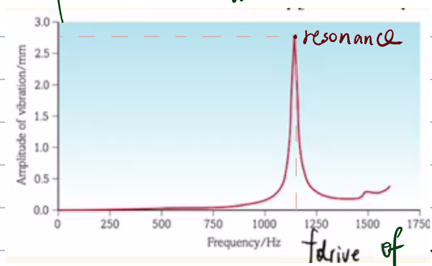
② Resonance 共振

条件: driving frequency (f_d) close to natural frequency (f_0)

↓

原因: external force do positive work to the system
↓
E of system
↓
结果: A ↑ with t

③ forced oscillation under different f (no damping)

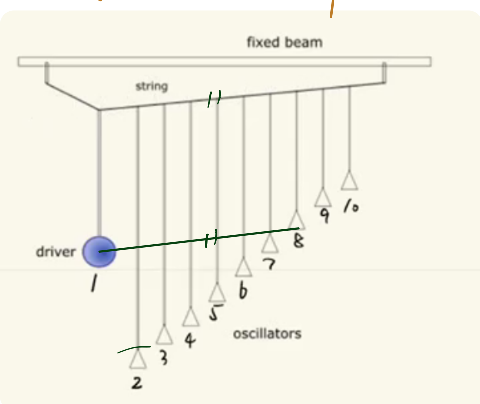


f_{drive} of external force

问题: resonance 在哪儿? Amplitude 最高处

问题: 其它 in f_d 的 oscillation 都怎么样?

3. 实验: Barron's pendulum



- What motion will the oscillators (1-10) do?
  1. damped oscillation - f_{10} ← l_1
  2-10. forced oscillation - $f_d = f_{10}$ ← l_1
- 2-10 有什么不同?
  - 8 (A_{max}) ⟵ $f_{80} = f_{10}$
  - others ($A \approx 0$) ⟵ $f_{n0} \neq f_{10}$

8 has greatest A, because $f_{08} \approx f_{driving} = f_{01}$, Energy accumulated, A can't increase

others: small A, because $f_{0n} \neq f_d = f_{01}$, energy can't accumulate

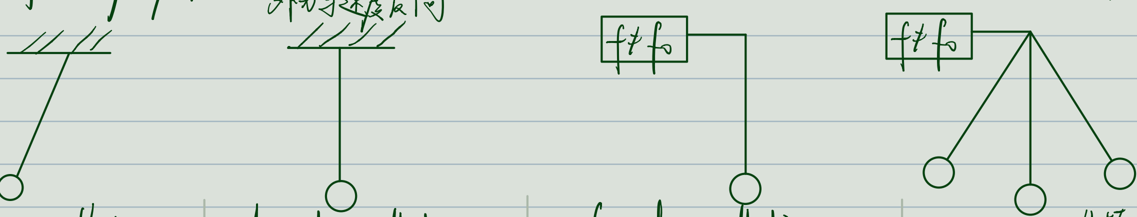
II. 辨析

1. 动手做

一个单摆, 如何演示振动?

外力与速度反向

Forced 的特殊形式



	free oscillation	damped oscillation	forced oscillation	resonance 共振
external force	$F_{ext}=0$ 只受保守力	continuous negative	periodic +/−	periodic positive
energy	constant	E↓ with t	$f_d=f_0$, E↑ with t $f_d \neq f_0$, no accumulate	E↑ with t
amplitude	constant	A↓ with t	$f_d=f_0$, A↑ with t $f_d \neq f_0$, no change	A↑ with t
frequency	f_0	f_0	f_d	constant = natural f

6.2.2 Resonance

I. Resonance in life

1. Resonance in machine

① 振源：engine, turbine, lathes
② 振体

⟹ f 达到一致时共振

2. Resonance in building

振源
振体

3. Resonance in electric circuit

LC 电路中电流

II. Solution of undesirable resonance

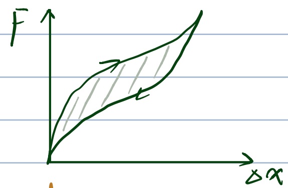
1. 条件 ($f_d \approx f_0$)　$f_0 = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

① make body stiffer / floppier
② add mass / remove

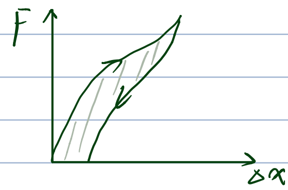
2. 能量 (damped forced oscillation)

① frictional force (E → heat)
② deformation (E → 内能) between 振源与振体

a. elastic:

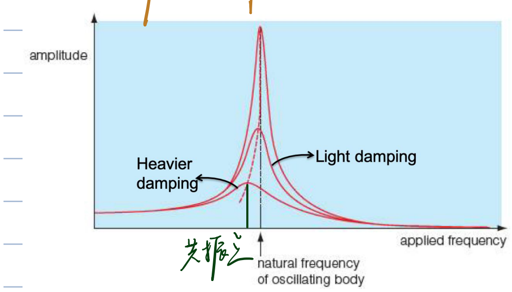


b. plastic.



deformation	优点
elastic	可重复
plastic	吸能效率高

III. Damped-forced oscillation



① y: $-F \xrightarrow{-W} E\downarrow \to A\downarrow$

② x: $f_{resonance} < f_0$
damping ↑ → Δf ↑

③ less damp

一. SHM

$\Updownarrow$ $\qquad \Sigma F = -\frac{mg}{l}\vec{x} \ (\theta < 10^\circ)$

$\Sigma F = -k\vec{x} \longrightarrow x = -\frac{k}{m}x \longrightarrow x = A\cos(\omega t + \varphi) \Rightarrow$

$\left\{\begin{array}{l} E_m = \frac{1}{2}kA^2 = \frac{1}{2}m\omega^2A^2 \\ E_p = \frac{1}{2}kx^2 = \frac{1}{4}kA^2(1+\cos 2(\omega t+\varphi)) \\ E_k = \frac{1}{2}kv^2 = \frac{1}{4}mA^2\omega^2(1-\cos 2(\omega t+\varphi)) \end{array}\right.$

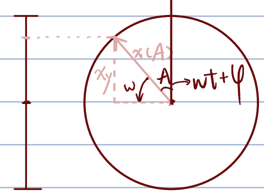
$a = -\omega^2 x \leftarrow \left\{\begin{array}{l} v = x' = -A\omega\sin(\omega t+\varphi) \\ a = x'' = -A\omega^2\cos(\omega t+\varphi) \end{array}\right.$

$\omega = \sqrt{\frac{k}{m}}$

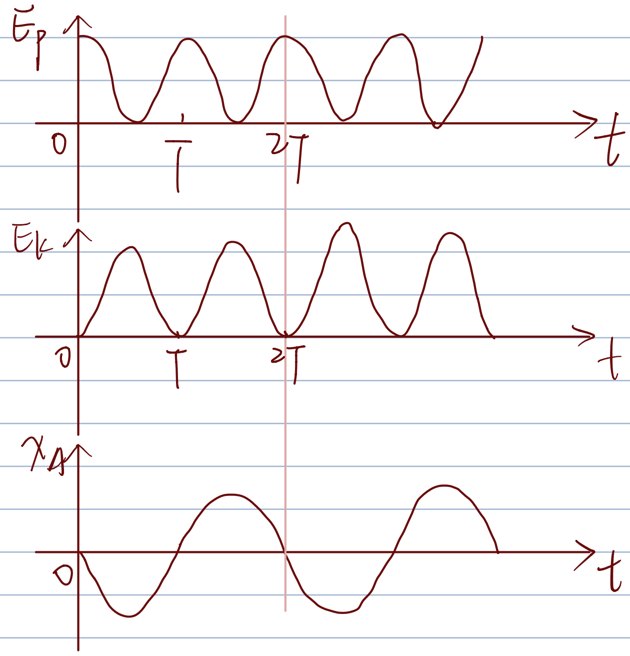
$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{l}{g}}$

$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$

$\left\{\begin{array}{l} x_{max} = A \\ v_{max} = A\omega \\ a_{max} = A\omega^2 \end{array}\right.$



- A : radius → amplitude
- ω : angular velocity → T
- φ : 起点



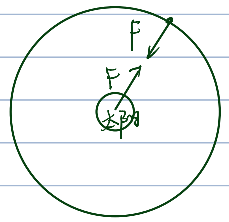
Oscillations	f of Fext	Fext 方向	Work	E/A
free	无	无	W=0	constant
damped	$f = f_0$	与v反向	W−	↓
forced	$f = f_0$	与v同向	W+	↑ resonance
	$f \neq f_0$	不相干	no accumulation	

7. Astrophysics & Cosmology

7.1 Gravitational fields

7.1.1 Gravitational forces

I. 万有引力定律



设行星做匀速直线运动

猜想 $\Sigma F_{星} \propto \frac{m}{r^2}$　$\Sigma F_{太阳} \propto \frac{M}{r^2}$

∵相互作用力　∴$\Sigma F \propto \frac{Mm}{r^2}$

$F = \frac{GMm}{r^2}$

$$F = \frac{GMm}{r^2} \quad (G = 6.67\times10^{-11}\,Nm^2/kg^2)$$

重力不是万有引力，G与万有引力的分力，为F支的平衡力（两极处，$F_引 = G$）

II. 天体称重

1. 地球

① $mg = G\frac{Mm}{R^2}$　$g = G\frac{M}{R^2} \Rightarrow M = \frac{gR^2}{G}$

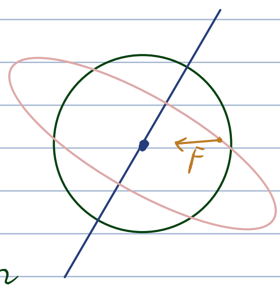
② 对月球　$G\frac{Mm}{r^2} = m\frac{4\pi^2}{T^2}r \Rightarrow M = \frac{4\pi^2 r^3}{GT^2}$　（r：地-月距离；T：月球公转周期；M：地球）

2. 太阳

对地球（绕太阳转）　$G\frac{Mm}{r^2} = m\frac{4\pi^2}{T^2}r \Rightarrow M = \frac{4\pi^2 r^3}{GT^2}$　（M：太阳；r：日-地距离）

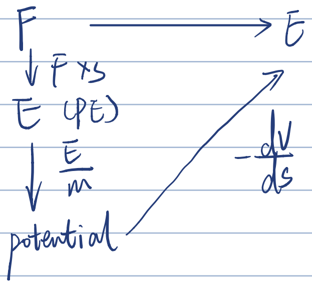
III. 同步卫星

1. 轨道：赤道
2. $\omega = \frac{2\pi}{24\times3600}$



$$\frac{GMm}{r^2} = m\frac{4\pi^2}{T^2}r \longrightarrow r = \sqrt[3]{\frac{GMT^2}{4\pi^2}} = 3.6\times10^7\,m$$

（T：自转周期）



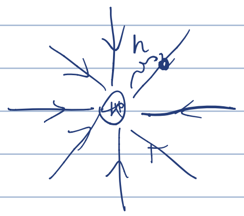
$V = -\frac{GM}{r}$

电势

$\Delta GPE = -W_F = \int F\,dx = \int_r^{\infty} \frac{GMm}{x^2}\,dx = [-GMm x^{-1}]_r^{\infty} = -\frac{GMm}{r}$

$V = \frac{\Delta GPE}{m} = -r$

①



$\Delta GPE = -W_F = -G\frac{Mm}{r^2}$

$= \int F\,dx$

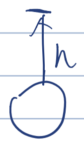
$= \left[-\frac{GMm}{r}\right]_R^{R+h}$ R：地球半径

$= \frac{GMmh}{(R+h)R} = \frac{GMm}{R^2}h = mgh$

$h \approx 0$

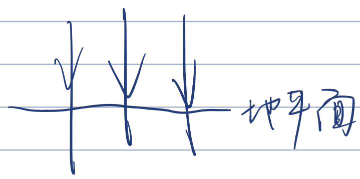
② potential difference $= \frac{\Delta GPE}{m} = \frac{-W_F}{m} = -\frac{GM}{r}$

③



$\Delta GPE = \frac{-GMm}{R+h} - \left(-\frac{GMm}{R}\right)$

④ Field



	Electric field	Gravitational field
源头	charge	mass
施加于	charge	mass
force laws	$F = \frac{kQ_1Q_2}{r^2}$	$F = \frac{GM_1M_2}{r^2}$
direction	同性相斥 异性相吸	always attracting
field strength	$E = \frac{F}{q}$ (NC^{-1})	$g = \frac{F}{m}$ (Nkg^{-1})
radial fields	$E = \frac{kQ}{r^2}$	$g = \frac{GM}{r^2}$
potential difference	$\Delta V = E\Delta x$ (JC^{-1})	$\Delta V = g\Delta h$

7.2 Astrophysics — Stars 星体

I. Radiation from stars

Luminosity: total output power of a star. (无法直接测出)

flux / intensity: 垂直穿过单位面积的 power

$$F = \frac{L}{4\pi d^2}$$ $J/s\,m^2$ 离 star d 米处接收到的

$J\,s^{-1}m^{-2} = W m^{-2}$

应该在大气层外观测
① 大气层会吸收部分波长
② 空气中的微粒会散质差

II. Black body (law 基于的假设)(理想化,不存在)

Good absorber: no radiation is reflected

good emitter: can emit electron-radiation in all frequencies

III. The Stefan-Boltzmann law

F $J s^{-1} m^{-2}$

d ↓

L $J s^{-1}$

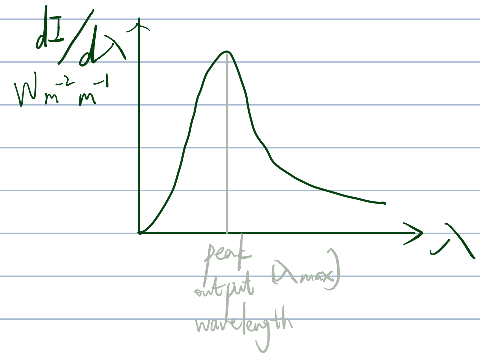
R ↓ 黑体假设

T

surface area $= 4\pi r^2$

Luminosity: $L = \sigma A T^4$

$\sigma = 5.67\times10^{-8}\ W m^{-2} K^{-4}$



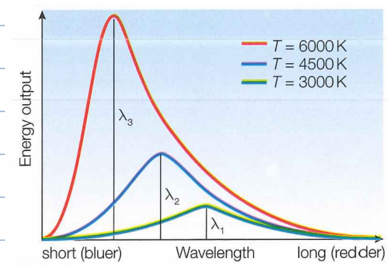
IV. Wien's law

T↑ → λ_{max}↓ → 颜色越红

$\lambda_{max} T = 2.898\times10^{-3}$ m K (meter-kelvin)

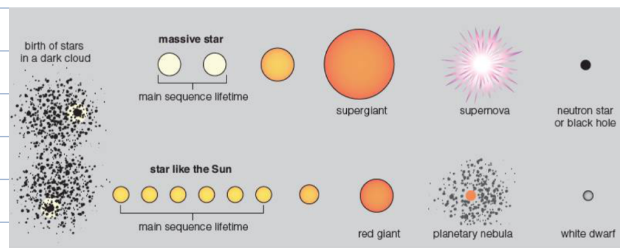
T —SB→ luminosity —($F = \frac{L}{4\pi d^2}$)→ flux

T —W→ λ_{max} ——→ color



V. The same formation of stars

① Nebula — protostar



② Main sequence (youth star, 90%)
- $T \approx 10^7 K$, $\rho \approx 10^5\ kg\,m^{-3}$, hydrogen fusion take place
- 平衡态: gravitational collapse ⟷ pressure from hydrogen fusion in the core

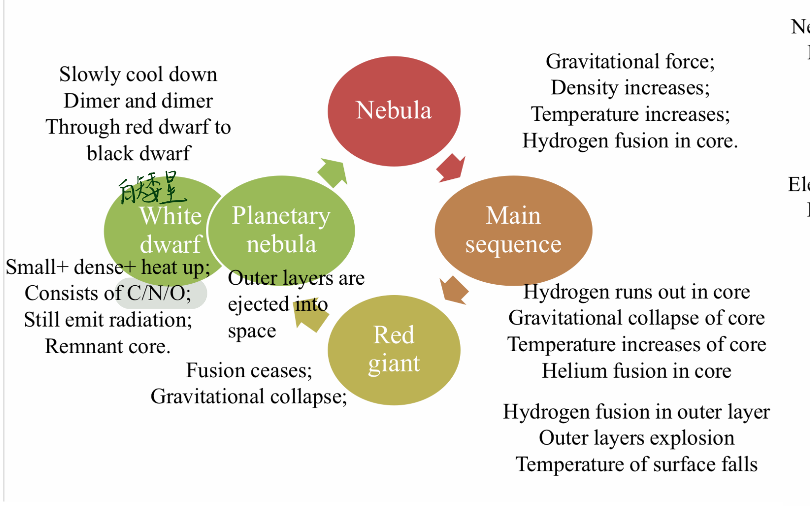
different ending
- $< 0.4 M$ — no main sequence
- $0.4 - 8 M$ — white dwarf
- $> 8 M$ — supernova

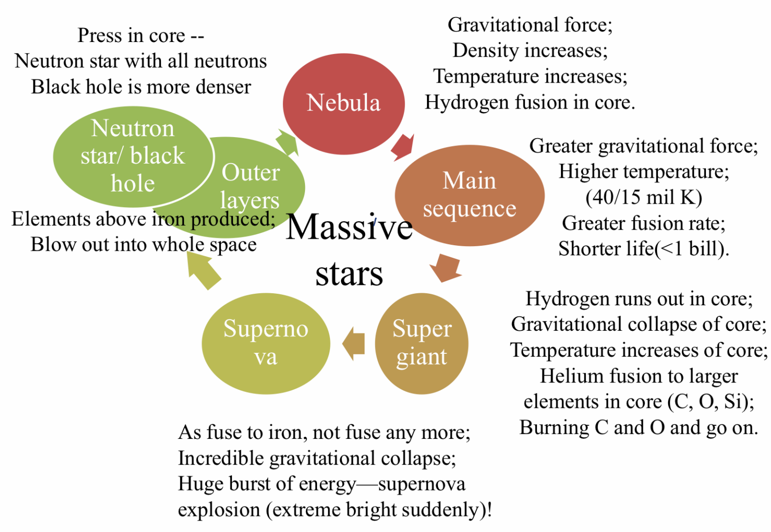
mass of remnant core
- $1.4 - 3 M_\odot$ — neutron stars
- $> 3 M_\odot$ — black holes

体积不会减小: $\Sigma F \neq 0 \rightarrow F_g > F_{fusion} \rightarrow V\downarrow \rightarrow \rho\uparrow, T\uparrow \rightarrow F_{fusion}\uparrow \rightarrow F_g = F_{fusion}$

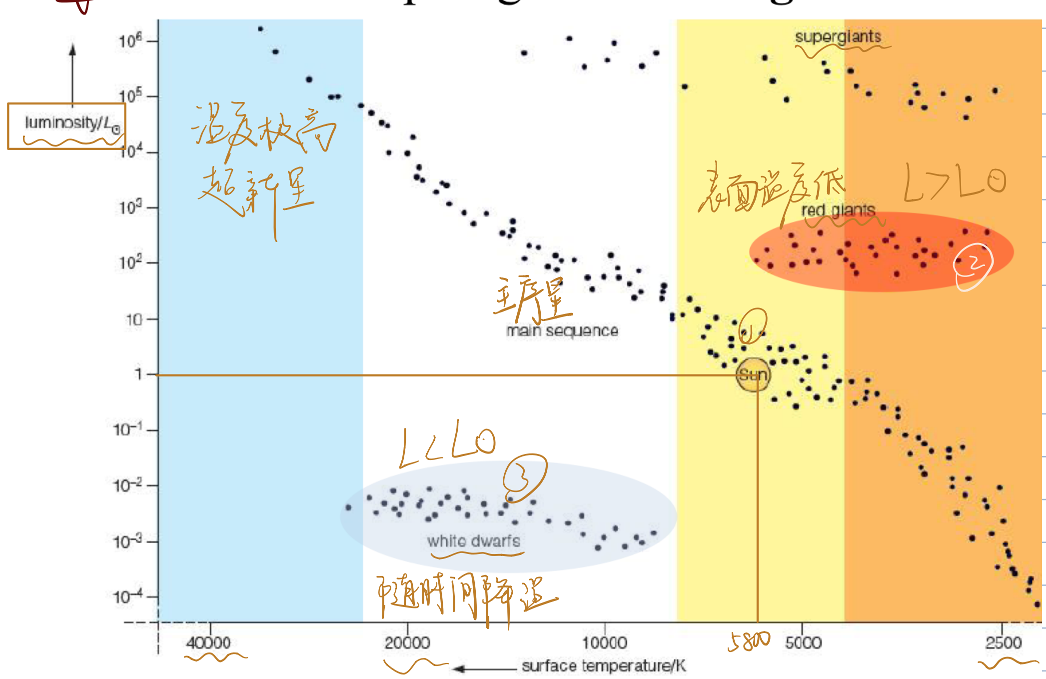
$T\uparrow \rightarrow \frac{1}{2}m\langle v^2\rangle\uparrow \rightarrow d$(间隔)$\downarrow \rightarrow$ fusion

For low-mass stars like the Sun





The Hertzsprung-Russell **diagram**



$L \propto T^4 A$

x, y轴: log scale

- It is a diagram not a graph – each dot represents a single star.
- The regions of main sequence stars, red giant stars and white dwarf stars
- Vertical axis:
  - luminosity of each star multiples of $L_\odot$ (no units)
  - Logarithmic scale, i.e. in powers of 10
- Horizontal axis:
  - Surface temperature T of the star in kelvin
  - Logarithmic scale, i.e. in powers of 2
  - From high temperature to low temperature
- The colours indicate what a star will look like

① The sun is fusing hydrogen.
When fusion stops, the temperature drops
The core collapse under gravatational force

② The sun expands to become red giant
The core becomes hot enough for helium fusion
Helium run out and core collapses again

③ outer layers of sun are ejected into space
the temperature can't rise to have further fusion

Another main sequence star, Gamma Pavonis, has a mass 20% greater than that of Delta Pavonis. It is suggested that Gamma Pavonis will stay on the main sequence for a greater time than Delta Pavonis.

Assess the validity of this suggestion.

m↑ → Fg↑ (引力) → P↑ → T↑ → rate of collision↑
↓
time↑

main sequence star: a star that is fusing hydrogen in its core

white dwarf star: star have small A. and have very high T to emit all visible λ.

7.3 Cosmology - Hubble's law 天文

* 如何测人到星星的距离 How to measure the distance of a star?

① λ_{max}

↓ Wien's law

Surface temperature

↓ H-R diagram

Flux + luminosity

↓

distance

} SB law → Radius

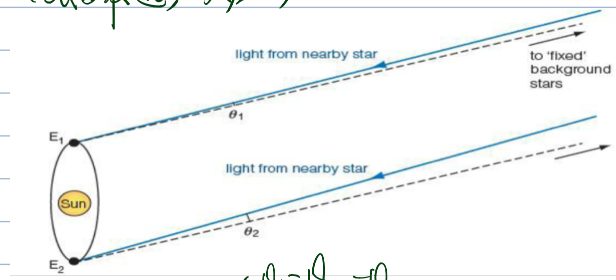
条件：仅限于主序星

限制：H-R图误差太大

② 视差法

取2个观测点 → 隔六个月测距离

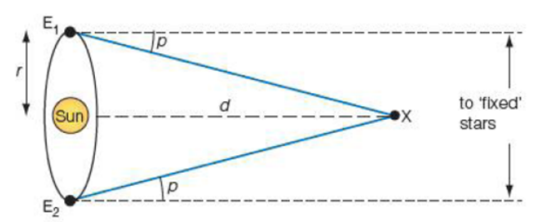
(距离最远，误差小)



$$\Delta\theta = \theta_2 - \theta_1$$

$$d = \frac{r}{\tan\left(\frac{\Delta\theta}{2}\right)} = \frac{2r}{\Delta\theta}$$

限制：无法精确测出很远的星星 (>100光年)



$$d = \frac{r}{tan(\Delta\theta/2)} = \frac{2r}{\Delta\theta}$$

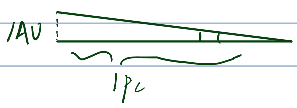
- For small angle: $\theta = \tan\theta = \sin\theta$
  - Unit of small angle: Second of arc ($1° = 60' = 3600''$: 1 degree = 60 minutes = 3600 seconds)
- **Parallax angle** ($\frac{\Delta\theta}{2}$) is the Earth-star-Sun angle which equals to half the measured **angle difference** ($\Delta\theta$)
- Parallax method only suitable for nearby star (<100 ly)

I. Big distance unit

- light year (ly): $1\,ly = 9.46 \times 10^{15}\,m$
- astronomical unit (AU): radius of the Earth's orbit around the sun 日地距离

  $1\,AU = 1.5 \times 10^{11}\,m$

  $1'' = \frac{1}{3600}°$
- Parsecs (pc): the distance of a star from the sun when the angle of Earth-star-Sun to be 1 arcsecond



$1\,parsec = 3.09 \times 10^{16}\,m = 3.26\,ly$

II. Standard candles 标准烛光 (已知luminosity的星星)

- 不稳定的 red giant
- 正在发生爆炸的 super giant

- def.

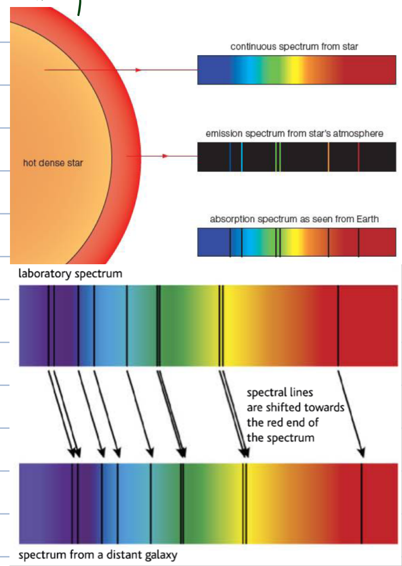
  a stellar object of known luminosity

— w to use a stan a can to measure diameter of a galaxy?

① standard candle has a known luminosity
② measure the brightness of the galaxy on earth
③ use $F = \frac{L}{4\pi d^2}$ to determine the distance

F: radiation flux on earth
L: luminosity of galaxy
d: distance from galaxy on earth

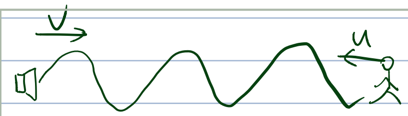
— 分析运动



IV. Doppler shift 多普勒效应

波源远离接收者 λ↑ f↓

relative movement of source and observer leads to a change in λ
+描述相对运动方向



$\lambda = \frac{v}{f}$ $\lambda = \frac{v+u}{f'}$

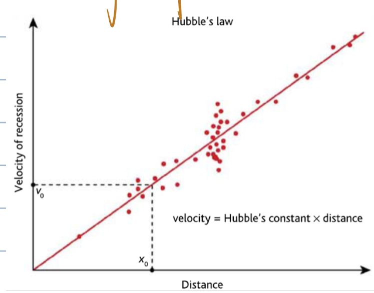
$\frac{v}{f} = \frac{v+u}{f'}$

$f' = \frac{v+u}{v} f$

— Red shift 红移现象
宇宙空间在扩大
吸收光谱因波源运动发生改变.

$z = \frac{\Delta\lambda}{\lambda} \approx \frac{v}{c}$
(v: 星体移动速度)

— The age of universe



$T = \frac{x_0}{v_0}$

From Hubble graph, $H_0 = \frac{v_0}{x_0}$

$T = \frac{1}{H_0}$
$= \frac{1}{2.31\times10^{-18} s^{-1}}$
$= 4.33\times10^{17} s$
$= 13.7$ *billion years*

$\Delta\lambda \xrightarrow{z = \frac{\Delta\lambda}{\lambda}} z \xrightarrow{z = \frac{v}{c}} v \xrightarrow{\frac{v}{d} = H_0} d$

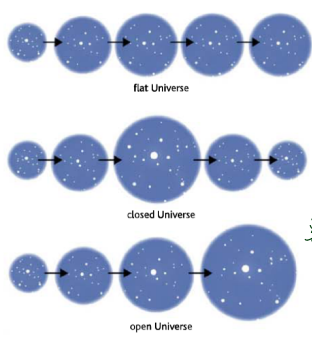
存在误差

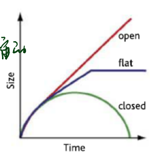
V. The flat universe 扩张/收缩

m↓ → ρ↓ → < ρ_c (critical density) 决定宇宙生命长度

closed $\rho > \rho_c$ gravitational force 大. 开始收缩
flat $\rho = \rho_c$ expand to max. 不收缩
open $\rho < \rho_c$ gravitational force too weak to stop expansion

How to determine ρ_c?





-The mass needed to keep the galaxy spinning is much greater than the mass of stars
-Only about 10% of matter is visible in the form of stars and gas clouds. The other 90% is called dark matter.
-Dark matter has mass/has gravitational effects but doesn't emit and interact with electromagnetic radiation.
-Estimate of mass is close to critical situation

1. Gravatational force

$\frac{GMm}{R^2} = m\omega^2 R$ 圆周

$\frac{GMm}{R^2} = ma$ 直线

} ← gravatational force → $g = \frac{F_g}{m} = \frac{GM}{r^2}$

↓

重力势能 $V_g = \frac{GPE}{m}$ ⇒ $\Delta V_g = -\vec{g} \cdot \Delta \vec{S}$

$\vec{g} = \frac{\Delta V_g}{S}$

球形场 $V_g = \frac{-GM}{r}$

$r \to \infty$ $V_g = 0$

2. Properties of stars

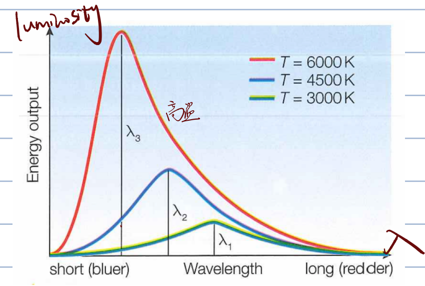
— Properties

flux $\xrightarrow{\times 4\pi d^2}$ luminosity ⟶ T

color ← λ_{max} $\xrightarrow[\lambda_{max}]{\text{Wien's}}$ T

$\Delta\lambda$ ⟶ velocity

(doppler shift)



— Evolution of stars

$m \to F_g \to P \to T$

↓

H fusion $\xrightarrow{\text{core}}$ Pressure in core = F_g main sequence

↓ H in core are exhausted red giant

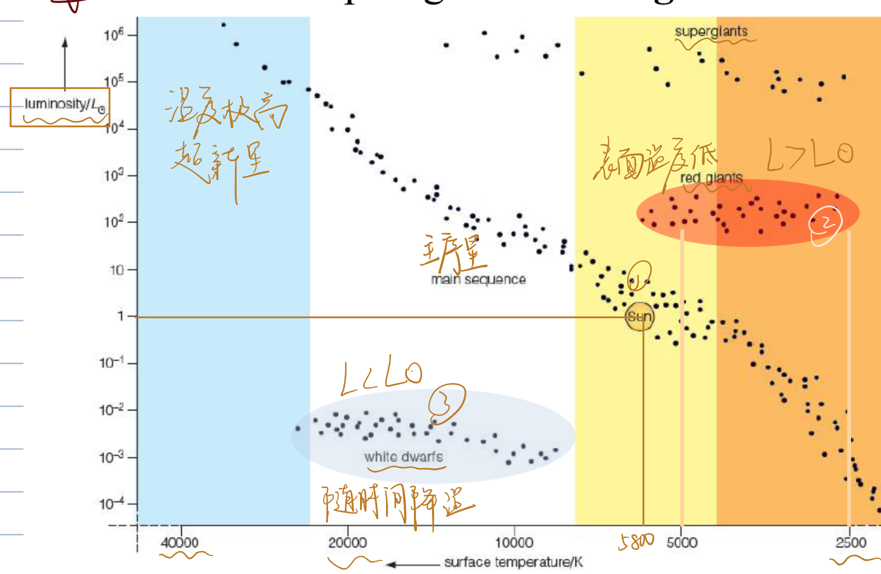
He fusion ← T_{core}↑ ← V↓ ← F_g 塌缩 → V_s↓ → T_s↑ ⟶ H fusion in axis → V↑ ⟶ T

↓ He in core exhaust

F_g ⟶ V↓ ⟶ T^2

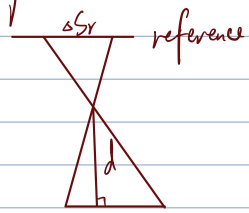
main red white

★ The Hertzsprung-Russell **diagram**



— Distance between stars

① parallax



② standard model

③ Doppler shift